$\mathcal{N} = 2$ Born-Infeld Theory with Auxiliary Superfields

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- Motivations: why Born-Infeld
- Self-duality: general setting
- Formulation with bispinor auxiliary fields
- $\mathcal{N} = 2$ BI theory: the $(\mathcal{W}, \bar{\mathcal{W}})$ formulation
- Auxiliary superfields for $\mathcal{N} = 2$ self-duality
- $(\mathcal{U}, \mathcal{W})$ form of the $\mathcal{N} = 2$ action up to 10th order
- Summary and Outlook

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Summary and Outlook

Motivations: why Born-Infeld

The BI theory is the renowned nonlinear extension of Maxwell theory:

$$L^{BI}(\varphi, \bar{\varphi}) = 1 - \sqrt{1 + \varphi + \bar{\varphi} + (1/4)(\varphi - \bar{\varphi})^2}\,,$$

$$\varphi + \bar{\varphi} = \frac{1}{2}(F_{mn})^2\,,\ \varphi - \bar{\varphi} = \frac{i}{2}F_{mn}\tilde{F}^{mn}.$$

Remarkable properties and applications:

- The BI theory is the first known example of nontrivial self-dual models of nonlinear electrodynamics;
- The BI actions are necessary ingredients of the static-gauge worldvolume actions of the D branes (non-abelian - in the general case of N coincident branes);
- The super D branes actions enjoy linearly realized worldvolume supersymmetries, and the relevant worldvolume supermultiplets are the vector ones, with the gauge field among the component fields. Thus the problem of constructing proper superextensions of the BI action is naturally posed;
- The spontaneously broken part of the full D brane supersymmetry should manifest itself as the second hidden nonlinear supersymmetry of these super BI actions.

To date, two superextended BI actions with the second nonlinearly realized supersymmetry are known in the superfield approach:

- $\mathcal{N}=1$ supersymmetric BI action (Cecotti & Ferrara 1987; Bagger & Galperin 1997). It describes the "space-filling" $D3$ brane and possesses the second nonlinear $\mathcal{N}=1$ supersymmetry building up the manifest one to $\mathcal{N}=2$. So it can be abbreviated as "$\mathcal{N}=2/\mathcal{N}=1$" BI action.
- $\mathcal{N}=2$ supersymmetric BI action (Bellucci, Ivanov, Krivonos 2000, 2001; Kuzenko, Theisen 2000, 2001). It describes the $D3$ brane in $D=6$ and possesses the nonlinearly realized $\mathcal{N}=4/\mathcal{N}=2$ supersymmetry, with $\mathcal{N}=4$ being properly extended by complex central charge generator.
- While the $\mathcal{N}=2/\mathcal{N}=1$ BI action is known in the closed superfield form, no such a formulation is known for the $\mathcal{N}=4/\mathcal{N}=2$ BI action so far. There exists another $\mathcal{N}=2$ BI action admitting the closed form (Ketov 1999), but it does not possess any extra supersymmetry.

It seems very interesting and urgent to further elaborate on the $\mathcal{N}=4/\mathcal{N}=2$ BI action and to try to find any closed form for it. There is one more intriguing problem related to this action.

- The $\mathcal{N}=2/\mathcal{N}=1$ BI action was shown to be self-dual like its bosonic counterpart. The same self-duality for the $\mathcal{N}=4/\mathcal{N}=2$ BI action was checked up to the 8th order in the $\mathcal{N}=2$ superfield strengths.
- The question is whether this self-duality property extends to all orders, and if does, what is the precise interplay between the self-duality and hidden supersymmetry.

- On the other hand, a new framework was recently developed for self-dual electrodynamics models and their $\mathcal{N} = 1, \mathcal{N} = 2$ extensions (Ivanov, Zupnik 2012; Kuzenko 2013; Ivanov, Lechtenfeld, Zupnik 2013). It proceeds from the formulation proposed in (Ivanov, Zupnik 2001, 2004) and includes, as a new ingredient, auxiliary tensorial fields and their superfield counterparts.
- It is natural to analyze the $\mathcal{N} = 4/\mathcal{N} = 2$ BI action within this general approach, in the hope that it will allow to give a general proof of self-duality of this action and, perhaps, to find an ansatz for the latter beyond the perturbative expansion over $\mathcal{N} = 2$ superfield strengths. The essential steps toward this goal was recently undertaken in Ivanov, Zupnik, arXiv:1312.5687 [hep-th]. Major part of my talk will be based on this work.

I'll start with recalling the salient features of self-duality in electrodynamics, then explain basics of the new approach with auxiliary tensorial fields, taking BI theory as an example. Then I'll discuss how to put the $\mathcal{N} = 4/\mathcal{N} = 2$ BI theory into this framework and to which new understanding it gives rise. Finally, I'll outline some perspectives.

Self-duality: general setting

$U(N)$ duality invariance: on-shell symmetry of a wide class of the nonlinear electrodynamics models including the renowned Born-Infeld theory

- Generalization of the free-case $O(2)$ symmetry between E.O.M. and Bianchi ($F_{mn} = \partial_m A_n - \partial_n A_m, \tilde{F}_{mn} = \frac{1}{2}\epsilon_{mnpq}F^{pq}$):

$$\text{E.O.M.} : \partial^m F_{mn} = 0 \qquad \Longleftrightarrow \qquad \text{Bianchi} : \partial^m \tilde{F}_{mn} = 0$$

$$\delta F_{mn} = \omega \tilde{F}_{mn}, \qquad \delta \tilde{F}_{mn} = -\omega F_{mn},$$

- In the nonlinear case: $P_{mn} = 2\frac{\partial L(F)}{\partial F^{mn}}$,

$$\text{E.O.M.} : \partial^m P_{mn} = 0 \qquad \Longleftrightarrow \qquad \text{Bianchi} : \partial^m \tilde{F}_{mn} = 0\,,$$

$$\delta P_{mn} = \omega \tilde{F}_{mn}\,, \qquad \delta \tilde{F}_{mn} = -\omega P_{mn}$$

.

- Self-consistency condition (so called GZ condition) (Galliard, Zumino, 1981; Gibbons, Rasheed, 1995):

$$P\tilde{P} - F\tilde{F} = 0$$

- Now - rebirth of interest in the duality-invariant theories and their superextensions (Kallosh, Bossard, Nicolai, Ferrara, ... , 2011, 2012,2013). The basic reason: generalized duality symmetry at quantum level can play the decisive role in proving the conjectured UV finiteness of $\mathcal{N} = 8, 4D$ supergravity!

More details

- We make use of the bispinor formalism, e.g. :

$$F_{mn} \Rightarrow (F_{\alpha\beta}, \quad \bar{F}_{\dot{\alpha}\dot{\beta}}), \quad \varphi = F^{\alpha\beta}F_{\alpha\beta}\,, \quad \bar{\varphi} = \bar{F}^{\dot{\alpha}\dot{\beta}}\bar{F}_{\dot{\alpha}\dot{\beta}}\,,$$

$$L(\varphi, \bar{\varphi}) = -\frac{1}{2}(\varphi + \bar{\varphi}) + L^{int}(\varphi, \bar{\varphi})$$

- E.O.M. and Bianchi:

$$\partial_{\alpha}^{\dot{\beta}}\bar{P}_{\dot{\alpha}\dot{\beta}}(F) - \partial_{\dot{\alpha}}^{\beta}P_{\alpha\beta}(F) = 0\,, \quad \partial_{\alpha}^{\dot{\beta}}\bar{F}_{\dot{\alpha}\dot{\beta}} - \partial_{\dot{\alpha}}^{\beta}F_{\alpha\beta} = 0\,, \quad P_{\alpha\beta} = i\frac{\partial L}{\partial F^{\alpha\beta}}$$

- $O(2)$ duality transformations

$$\delta_\omega F_{\alpha\beta} = \omega P_{\alpha\beta}, \qquad \delta_\omega P_{\alpha\beta} = -\omega F_{\alpha\beta},$$

- The self-consistency GZ constraint and the GZ representation for L:

$$F_{\alpha\beta}F^{\alpha\beta} + P_{\alpha\beta}P^{\alpha\beta} - \text{c.c} = 0 \quad \Leftrightarrow \quad \varphi - \bar{\varphi} - 4\,[\varphi(L_\varphi)^2 - \bar{\varphi}(L_{\bar{\varphi}})^2] = 0\,,$$

$$L^{sd} = \frac{i}{2}(\bar{P}\bar{F} - PF) + I(\varphi, \bar{\varphi})\,, \quad \delta_\omega I(\varphi, \bar{\varphi}) = 0$$

- How to determine $I(\varphi, \bar{\varphi})$? Our approach with the auxiliary tensorial fields provides an answer.

Formulation with bispinor auxiliary fields

- Introduce the auxiliary unconstrained fields $V_{\alpha\beta}$ and $\bar{V}_{\dot{\alpha}\dot{\beta}}$ and write the extended Lagrangian in the (F, V)-representation as

$$\mathcal{L}(V,F) = \mathcal{L}_2(V,F) + E(\nu,\bar{\nu}), \quad \mathcal{L}_2(V,F) = \frac{1}{2}(\varphi+\bar{\varphi}) + \nu + \bar{\nu} - 2\,(V\cdot F + \bar{V}\cdot\bar{F}),$$

  with $\nu = V^2, \ \bar{\nu} = \bar{V}^2$. Here $\mathcal{L}_2(V,F)$ is the bilinear part only through which the Maxwell field strength enters the action and $E(\nu,\bar{\nu})$ is the nonlinear interaction involving only auxiliary fields.
- Dynamical equations of motion

$$\partial^{\dot{\beta}}_{\alpha}\bar{P}_{\dot{\alpha}\dot{\beta}}(V,F) - \partial^{\beta}_{\dot{\alpha}}P_{\alpha\beta}(V,F) = 0\,, \quad P_{\alpha\beta}(F,V) = i\frac{\partial\mathcal{L}(V,F)}{\partial F^{\alpha\beta}} = i(F_{\alpha\beta} - 2V_{\alpha\beta}),$$

  together with Bianchi identity, are covariant under $O(2)$ transformations

$$\delta V_{\alpha\beta} = -i\omega V_{\alpha\beta}, \ \delta F_{\alpha\beta} = i\omega(F_{\alpha\beta} - 2V_{\alpha\beta}), \ \ \delta\nu = -2i\omega\nu$$

- Algebraic equation of motion for $V_{\alpha\beta}$,

$$F_{\alpha\beta} = V_{\alpha\beta} + \frac{1}{2}\frac{\partial E}{\partial V^{\alpha\beta}} = V_{\alpha\beta}(1 + E_\nu)\,,$$

  is $O(2)$ covariant if and only if the proper constraint holds:

$$\nu E_\nu - \bar{\nu}E_{\bar{\nu}} = 0 \quad \Rightarrow \quad E(\nu,\bar{\nu}) = \mathcal{E}(a)\,, \ \ a := \nu\bar{\nu}$$

- The meaning of this constraint - $E(\nu,\bar\nu)$ should be $O(2)$ invariant function of the auxiliary tensor variables

$$\delta_\omega E = 2i\omega(\bar\nu E_{\bar\nu} - \nu E_\nu) = 0$$

This is none other than the GZ constraint in the new setting:

$$F^2 + P^2 - \bar F^2 - \bar P^2 = 0 \quad \Longleftrightarrow \quad \nu E_\nu - \bar\nu E_{\bar\nu} = 0$$

- The auxiliary equation can be now written as

$$F_{\alpha\beta} - V_{\alpha\beta} = V_{\alpha\beta}\bar\nu\,\mathcal{E}'$$

It and its conjugate serve to express $V_{\alpha\beta}, \bar V_{\dot\alpha\dot\beta}$ in terms of $F_{\alpha\beta}, \bar F_{\dot\alpha\dot\beta}$:

$$V_{\alpha\beta}(F) = F_{\alpha\beta}G(\varphi,\bar\varphi), \quad G(\varphi,\bar\varphi) = \frac{1}{2} - L_\varphi = (1+\bar\nu\mathcal{E}_a)^{-1}$$

After substituting these expressions back into $\mathcal{L}$ we obtain the corresponding selfdual $L(\varphi,\bar\varphi)$

$$L^{sd}(\varphi,\bar\varphi) = \mathcal{L}(V(F),F) = -\frac{1}{2}\,\frac{(\varphi+\bar\varphi)(1-a\mathcal{E}_a^2) + 8a^2\mathcal{E}_a^3}{1+a\mathcal{E}_a^2} + \mathcal{E}(a)$$

where a is related to $\varphi,\bar\varphi$ by the algebraic equation

$$(1+a\mathcal{E}_a^2)^2\varphi\bar\varphi = a[(\varphi+\bar\varphi)\mathcal{E}_a + (1-a\mathcal{E}_a^2)^2]^2$$

- The invariant GZ function: $I(\varphi,\bar\varphi) = \mathcal{E}(a) - 2a\mathcal{E}_a$, $a = \nu(\varphi,\bar\varphi)\bar\nu(\varphi,\bar\varphi)$.

To summarize, *all* $O(2)$ duality-symmetric systems of nonlinear electrodynamics without derivatives on the field strengths are parametrized by the $O(2)$ invariant off-shell interaction $\mathcal{E}(a)$ which is a function of the real quartic combination of the auxiliary fields. This universality is the basic advantage of the approach with tensorial auxiliary fields. Actually, it works perfectly also for self-dual actions with higher derivatives.

After passing to the tensorial notation, our basic auxiliary field equation (proposed ten years ago) precisely coincides with what was recently called "nonlinear twisted self-duality constraint" (Bossard & Nicolai, 2011; Carrasco, Kallosh & Roiban, 2012; Chemissany, Kallosh & Ortin, 2012) and $\mathcal{E}(a)$ with what is called there "duality invariant source of deformation".

Example: Born-Infeld through auxiliary fields

The BI model has a more simple description in terms of the new variables:

$$\mathcal{E}^{BI}(a) = I^{BI}(b) - 2b\, I_b^{BI}(b)\,, \quad I^{BI} = \frac{2b}{b-1},\; I_b^{BI} = -\frac{2}{(b-1)^2}\,,\; a = \frac{4b}{(1-b)^4}.$$

The equation for computing b becomes quadratic:

$$\varphi\bar{\varphi}\, b^2 + [2\varphi\bar{\varphi} - (\varphi + \bar{\varphi} + 2)^2]\, b + \varphi\bar{\varphi} = 0 \qquad \Rightarrow$$

$$b = \frac{4\varphi\bar{\varphi}}{[2(1+Q) + \varphi + \bar{\varphi}]^2}\,, \quad Q(\varphi) = \sqrt{1 + \varphi + \bar{\varphi} + (1/4)(\varphi - \bar{\varphi})^2}$$

After substituting this into the general formula for $L^{cd}(\varphi, \bar{\varphi})$ the standard BI Lagrangian is recovered

$$L^{BI}(\varphi, \bar{\varphi}) = 1 - \sqrt{1 + \varphi + \bar{\varphi} + (1/4)(\varphi - \bar{\varphi})^2}\,.$$

How to supersymmetrize the bispinor formulation?

- The basic idea (Kuzenko, 2013; Ivanov, Lechtenfeld, Zupnik, 2013): to embed tensorial auxiliary fields into chiral auxiliary $\mathcal{N} = 1, 2$ superfields:

$$V_{\alpha\beta}(x) \;\Rightarrow\; U_\alpha(x, \theta, \bar\theta) = v_\alpha(x) + \theta^\beta V_{\alpha\beta}(x) + \ldots, \;\; \bar D_{\dot\gamma} U_\alpha(x, \theta, \bar\theta) = 0$$

$$V_{\alpha\beta}(x) \;\Rightarrow\; U(x, \theta^i, \bar\theta_i) = v(x) + (\theta^\beta_k \theta^{\alpha k}) V_{\alpha\beta}(x) + \ldots, \;\; \bar D_{\dot\gamma i} U(x, \theta^i, \bar\theta_k) = 0\,.$$

- The (W, U) representation for the $\mathcal{N} = 1$ self-dual actions:

$$S(W, U) = \int d^6\zeta \left(UW - \frac{1}{2}U^2 - \frac{1}{4}W^2 \right) + \text{c.c.} + \frac{1}{4}\int d^8 z\, U^2 \bar U^2\, E(u, \bar u, g, \bar g),$$

$$u = \frac{1}{8}\bar D^2 \bar U^2, \quad \bar u = \frac{1}{8} D^2 U^2, \quad g = D^\alpha U_\alpha\,.$$

- Duality-invariant $\mathcal{N} = 1$ systems amount to the $U(1)$ invariant interaction

$$E_{inv} = \mathcal{F}(B, A, C) + \bar{\mathcal{F}}(\bar B, A, C)\,, \quad A := u\bar u\,,\; C := g\bar g\,,\; B := ug^2\,,\; \bar B := \bar u \bar g^2\,.$$

$\mathcal{N}=2$ BI theory: the $(\mathcal{W},\bar{\mathcal{W}})$ formulation

The $\mathcal{N}=2$ BI action can be written through the chiral Lagrangian density $\mathcal{A}_0$ as (Bellucci, Ivanov, Krivonos 2001, hep-th/0012236,0101195)

$$S_{BI}(\mathcal{W}) = S_2(\mathcal{W}) + I_{BI}(\mathcal{W}) = \frac{1}{4}\int d^8\mathcal{Z}\,\mathcal{A}_0 + \frac{1}{4}\int d^8\bar{\mathcal{Z}}\,\bar{\mathcal{A}}_0\,,$$

$$I_{BI}(\mathcal{W}) = \int d^{12}Z\,L_{BI}(\mathcal{W})\,,$$

where

$$L_{BI} = \sum_{n=2}^{\infty} L^{(2n)}, \quad \mathcal{A}_0(\mathcal{W}) = \sum_{n=1}^{\infty}\mathcal{A}_0^{(2n)} = \mathcal{W}^2 + 2\bar{D}^4 L_{BI}\,.$$

The superfield strength $\mathcal{W}$, together with $\mathcal{A}_0$, belong to an infinite-dimensional multiplet of the spontaneously broken $\mathcal{N}=4$ supersymmetry

$$\delta_f\mathcal{W} = f(1-\frac{1}{2}\bar{D}^4\bar{\mathcal{A}}_0) + \frac{1}{2}\bar{f}\Box\mathcal{A}_0 + \frac{i}{4}\bar{D}^{\dot\alpha}_k\bar{f}D^{k\alpha}\partial_{\alpha\dot\alpha}\mathcal{A}_0,$$

$$\delta_f\mathcal{A}_0 = 2f\mathcal{W} + \frac{1}{2}\bar{f}\Box\mathcal{A}_1 + \frac{i}{4}\bar{D}^{\dot\alpha}_k\bar{f}D^{k\alpha}\partial_{\alpha\dot\alpha}\mathcal{A}_1\,,$$

$$\delta_f\mathcal{A}_n = 2f\mathcal{A}_{n-1} + \frac{1}{2}\bar{f}\Box\mathcal{A}_{n+1} + \frac{i}{4}\bar{D}^{\dot\alpha}_k\bar{f}D^{k\alpha}\partial_{\alpha\dot\alpha}\mathcal{A}_{n+1}\,,\ n\geq 1\,,$$

$$f = c + 2i\theta^\alpha_k\xi^k_\alpha\,.$$

The density $\mathcal{A}_0$, together with other superfields $\mathcal{A}_n$, can be expressed in terms of $(\mathcal{W}, \bar{\mathcal{W}})$ by imposing an infinite number of $\mathcal{N} = 4$ covariant constraints

$$\mathcal{A}_0 - \mathcal{W}^2 - \frac{1}{2}\mathcal{A}_0 \bar{D}^4 \bar{\mathcal{A}}_0 - \bar{D}^4 \sum_{n=1} \frac{(-1)^n}{2^{2n+1}} \mathcal{A}_n \Box^n \bar{\mathcal{A}}_n = 0\,,$$

$$\Box \mathcal{A}_1 + 2(\mathcal{A}_0 \Box \mathcal{W} - \mathcal{W} \Box \mathcal{A}_0) - \bar{D}^4 \sum_{n=0} \frac{(-1)^n}{2^{2n+1}} (\Box \mathcal{A}_{n+1} \Box^n \bar{\mathcal{A}}_n - \mathcal{A}_{n+1} \Box^{n+1} \bar{\mathcal{A}}_n) = 0\,,$$

$$\Box^2 \mathcal{A}_2 + 2(\mathcal{A}_0 \Box^2 \mathcal{A}_0 - \Box \mathcal{A}_0 \Box \mathcal{A}_0 + 2 \Box \mathcal{W} \Box \mathcal{A}_1 - \mathcal{W} \Box^2 \mathcal{A}_1 - \mathcal{A}_1 \Box^2 \mathcal{W})$$

$$- \bar{D}^4 \sum_{n=0} \frac{(-1)^n}{2^{2n+1}} (\Box^2 \mathcal{A}_{n+2} \Box^n \bar{\mathcal{A}}_n - 2 \Box \mathcal{A}_{n+2} \Box^{n+1} \bar{\mathcal{A}}_n + \mathcal{A}_{n+2} \Box^{n+2} \bar{\mathcal{A}}_n) = 0\,,$$

$$\cdots\cdots\cdots$$

$$\Box^n \mathcal{A}_n + \ldots = 0$$

Solving them by recursions, one can restore $\mathcal{A}_0$ to any order in $\mathcal{W}, \bar{\mathcal{W}}$, e.g.,

$$\mathcal{A}_0^{(4)} = \frac{1}{2} \bar{D}^4 (\mathcal{W}^2 \bar{\mathcal{W}}^2), \quad \mathcal{A}_0^{(6)} = \frac{1}{4} \bar{D}^4 \left[\mathcal{W}^2 \bar{\mathcal{W}}^2 (D^4 \mathcal{W}^2 + \bar{D}^4 \bar{\mathcal{W}}^2) - \frac{2}{9} \mathcal{W}^3 \Box \bar{\mathcal{W}}^3 \right]$$

In fact, as shown in Ivanov & Zupnik, 2013, there is an alternative way of restoring $\mathcal{A}_0$, based solely upon the nonlinear realization of the central charge generators on $\mathcal{W}, \bar{\mathcal{W}}$.

In this way, the interaction I_{BI} was found in hep-th/0101195 up to the 8th order

$$I^{(4)}_{BI} = \frac{1}{4}\int d^{12}Z\,\mathcal{W}^2\bar{\mathcal{W}}^2\,,$$

$$I^{(6)}_{BI} = \frac{1}{8}\int d^{12}Z\Big[\mathcal{W}^2\bar{\mathcal{W}}^2(D^4\mathcal{W}^2 + \bar{D}^4\bar{\mathcal{W}}^2) - \frac{2}{9}\mathcal{W}^3\Box\bar{\mathcal{W}}^3\Big],$$

$$I^{(8)}_{BI} = \frac{1}{16}\int d^{12}Z\Big\{\mathcal{W}^2\bar{\mathcal{W}}^2[(D^4\mathcal{W}^2)^2 + (\bar{D}^4\bar{\mathcal{W}}^2)^2 + 3D^4\mathcal{W}^2\bar{D}^4\bar{\mathcal{W}}^2]$$
$$- \frac{2}{3}[\bar{\mathcal{W}}^3 D^4\mathcal{W}^2\Box\mathcal{W}^3 + \mathcal{W}^3\bar{D}^4\bar{\mathcal{W}}^2\Box\bar{\mathcal{W}}^3] + \frac{1}{36}\mathcal{W}^4\Box^2\bar{\mathcal{W}}^4\Big\}.$$

Recently (Ivanov & Zupnik, 1312.5687 [hep-th]), the next, 10th order term, was explicitly computed

$$I^{(10)}_{BI} = \frac{1}{8}\int d^{12}Z\Big\{\frac{1}{4}\mathcal{W}^2\bar{\mathcal{W}}^2\Big[(D^4\mathcal{W}^2)^3 + (\bar{D}^4\bar{\mathcal{W}}^2)^3 + 4(D^4\mathcal{W}^2)^2\bar{D}^4\bar{\mathcal{W}}^2$$
$$+ 4D^4\mathcal{W}^2(\bar{D}^4\bar{\mathcal{W}}^2)^2 + 2\bar{D}^4\bar{\mathcal{W}}^2 D^4(\mathcal{W}^2\bar{D}^4\bar{\mathcal{W}}^2) + 2D^4\mathcal{W}^2\bar{D}^4(\bar{\mathcal{W}}^2 D^4\mathcal{W}^2)\Big]$$
$$-\frac{1}{3}\mathcal{W}^3\bar{\mathcal{W}}^2\bar{D}^4\bar{\mathcal{W}}^2\Box\bar{D}^4\bar{\mathcal{W}}^3 - \frac{1}{3}\mathcal{W}^2\bar{\mathcal{W}}^3 D^4\mathcal{W}^2\Box D^4\mathcal{W}^3$$
$$-\frac{2}{9}\mathcal{W}^3\bar{\mathcal{W}}^2 D^4\mathcal{W}^2\Box\bar{D}^4\bar{\mathcal{W}}^3 - \frac{2}{9}\mathcal{W}^2\bar{\mathcal{W}}^3\bar{D}^4\bar{\mathcal{W}}^2\Box D^4\mathcal{W}^3 - \frac{4}{9}\mathcal{W}^3\bar{D}^4\bar{\mathcal{W}}^2\Box(\bar{\mathcal{W}}^3 D^4\mathcal{W}^2)$$
$$+ \frac{1}{36}\mathcal{W}^4\Box\bar{\mathcal{W}}^3\Box\bar{D}^4\bar{\mathcal{W}}^3 + \frac{1}{36}\bar{\mathcal{W}}^4\Box\mathcal{W}^3\Box D^4\mathcal{W}^3$$
$$+ \frac{1}{36}\mathcal{W}^4\bar{D}^4\bar{\mathcal{W}}^2\Box^2\bar{\mathcal{W}}^4 + \frac{1}{36}\bar{\mathcal{W}}^4 D^4\mathcal{W}^2\Box^2\mathcal{W}^4 - \frac{1}{1800}\mathcal{W}^5\Box^3\bar{\mathcal{W}}^5\Big\}.$$

Inspecting the explicit structure of the perturbative terms in $\mathcal{A}_0$ and the respective terms in I_{BI}, we found that $\mathcal{A}_0$ has the following general splitting

$$\mathcal{A}_0 = \mathcal{X} + \mathcal{R} + \mathcal{Y}.$$

Here, $\mathcal{X}$ is defined by the equation (Ketov 1999, Kuzenko & Theisen 2000, 2001)

$$\mathcal{X} = \mathcal{W}^2 + \frac{1}{2}\mathcal{X}\bar{D}^4\bar{\mathcal{X}}$$

and accounts for all terms without box operators $\Box$. The part $\mathcal{R}$ accounts for all terms containing only box operators

$$\mathcal{R} = 2\bar{D}^4 \sum_{n=3}^{\infty}(-1)^n \frac{1}{(n!)^2}\mathcal{W}^n \Box^{n-2}\bar{\mathcal{W}}^n.$$

The remaining piece $\mathcal{Y}$ collects, in its perturbative expansion, the mixed terms with $D^4, \bar{D}^4, \Box$, which are not combined into any obvious series. It contributes to the interaction I_{BI}, starting from the 8th order.

Is it possible to give some general formulas for all these pieces? Is it possible to prove, at least up to the 10th order, that the $\mathcal{N} = 4/\mathcal{N} = 2$ BI action is simultaneously self-dual? The latter property was proved by Kuzenko & Theisen up to the 8th order. The reformulation of the $\mathcal{N} = 2$ BI theory through auxiliary superfields turns out to be helpful for getting the answers.

Auxiliary superfields for $\mathcal{N} = 2$ self-duality

With the auxiliary chiral scalar superfield $\mathcal{U}$, the $(\mathcal{W}, \mathcal{U})$ representation of the general action of the nonlinear $\mathcal{N} = 2$ electrodynamics reads

$$\mathcal{S}(\mathcal{W}, \mathcal{U}) = \mathcal{S}_b(\mathcal{W}, \mathcal{U}) + \mathcal{I}(\mathcal{U}) \,, \quad \mathcal{I}(\mathcal{U}) = \int d^8\mathcal{Z}\, \mathcal{L}(\mathcal{U}) + \text{c.c.}\,,$$

$$\mathcal{S}_b(\mathcal{W}, \mathcal{U}) = \int d^8\mathcal{Z}\, \mathcal{L}_b(\mathcal{W}, \mathcal{U}) + \text{c.c.}\,, \quad \mathcal{L}_b(\mathcal{W}, \mathcal{U}) = -\frac{1}{2}\mathcal{U}^2 + \mathcal{U}\mathcal{W} - \frac{1}{4}\mathcal{W}^2\,.$$

The interaction $\mathcal{L}(\mathcal{U})$ is a local function of $\mathcal{U}, \bar{\mathcal{U}}$ and all superfields one can construct from $\mathcal{U}, \bar{\mathcal{U}}$ and their x and θ derivatives. The $\mathcal{U}$ equation

$$\mathcal{U} = \mathcal{W} + \frac{\delta\mathcal{I}}{\delta\mathcal{U}}\,, \qquad \frac{\delta\mathcal{I}}{\delta\mathcal{U}} := \mathcal{J}(\mathcal{U}, \bar{\mathcal{U}}) = \bar{D}^4 J(\mathcal{U}, \bar{\mathcal{U}})\,, \tag{1}$$

allows one to eliminate auxiliary superfields and to recover the nonlinear $\mathcal{W}$ action. The $\mathcal{N} = 2$ self-duality condition (Kuzenko & Theisen 2000) becomes

$$\int d^8\mathcal{Z}(\mathcal{W}\mathcal{U} - \mathcal{U}^2) = \int d^8\bar{\mathcal{Z}}(\bar{\mathcal{W}}\bar{\mathcal{U}} - \bar{\mathcal{U}}^2)\,,$$

and is further reduced to

$$\int d^8\mathcal{Z}\, \mathcal{U}\frac{\delta\mathcal{I}}{\delta\mathcal{U}} = \int d^8\bar{\mathcal{Z}}\, \bar{\mathcal{U}}\frac{\delta\mathcal{I}}{\delta\bar{\mathcal{U}}}\,.$$

This is just the condition of the invariance of the functional $\mathcal{I}(\mathcal{U})$ under the $U(1)$ transformations $\delta_\omega\mathcal{U} = -i\omega\mathcal{U}\,,\ \delta_\omega\bar{\mathcal{U}} = i\omega\bar{\mathcal{U}}$.

Thus any self-dual system of $\mathcal{N}=2$ electrodynamics can be reformulated as a system with the off-shell action $\mathcal{S}(\mathcal{W},\mathcal{U})=\mathcal{S}_b(\mathcal{W},\mathcal{U})+\mathcal{I}(\mathcal{U})$, in which the interaction $\mathcal{I}(\mathcal{U})$ is invariant under the above $U(1)$ transformations.

Conversely, if some $\mathcal{N}=2$ system **admits** such a reformulation, **it is self-dual**.

One of the ways to prove that the $\mathcal{N}=4/\mathcal{N}=2$ BI action is self-dual is to put it into the $(\mathcal{U},\mathcal{W})$ form and to show that $\mathcal{I}_{BI}(\mathcal{U})$ is $U(1)$ invariant.

So the goal is to write the $\mathcal{N}=4/\mathcal{N}=2$ BI action as

$$\mathcal{S}_{BI}(\mathcal{W},\mathcal{U})=\mathcal{S}_b(\mathcal{W},\mathcal{U})+\mathcal{I}_{BI}(\mathcal{U})=\int d^8\mathcal{Z}\,\mathcal{L}_{BI}(\mathcal{W},\mathcal{U})+\text{c.c.}\,,$$

where, in accordance with the triple decomposition of the $(\mathcal{W},\bar{\mathcal{W}})$ BI action,

$$\mathcal{I}_{BI}(\mathcal{U})=\mathcal{I}_{\mathcal{X}}(\mathcal{U})+\mathcal{I}_{\mathcal{R}}(\mathcal{U})+\mathcal{I}_{\mathcal{Y}}(\mathcal{Y})\,.$$

The term $\mathcal{I}_{\mathcal{X}}(\mathcal{U})$ in the absence of other terms should generate the action associated with the chiral superfield $\mathcal{X}$ defined above (Ketov 1999). It is known that it is self-dual (Kuzenko & Theisen). The term

$$\mathcal{I}_{\mathcal{R}}(\mathcal{U})=\int d^8\mathcal{Z}\,\mathcal{L}_{\mathcal{R}}(\mathcal{U})+\text{c.c.}\,,\quad \mathcal{L}_{\mathcal{R}}=\frac{1}{2}\bar{D}^4\sum_{n=3}^{\infty}(-1)^n\frac{1}{(n!)^2}\mathcal{U}^n\Box^{n-2}\bar{\mathcal{U}}^n$$

is the only structure capable to reproduce the highest-derivative $\mathcal{R}$ contributions in the $\mathcal{W}$ representation. At last, $\mathcal{I}_{\mathcal{Y}}(\mathcal{U})$ stands for possible corrections to the contributions of the first two terms.

The term $\mathcal{I}_{\mathcal{X}}$

The term $\mathcal{I}_{\mathcal{X}}(\mathcal{U})$ should reproduce the nonlinear action

$$S_{\mathcal{X}}(\mathcal{W}) = \frac{1}{4}\int d^8\mathcal{Z}\,\mathcal{X}(\mathcal{W}) + \frac{1}{4}\int d^8\bar{\mathcal{Z}}\,\bar{\mathcal{X}}(\mathcal{W})\,, \quad \mathcal{X} = \mathcal{W}^2 + \frac{1}{2}\mathcal{X}\bar{D}^4\bar{\mathcal{X}}.$$

After some work with introducing auxiliary superfields, this action can be equivalently represented as

$$\mathcal{S}_{\mathcal{X}}(\mathcal{W},\mathcal{U},N) = \mathcal{S}_b(\mathcal{W},\mathcal{U}) + \mathcal{I}_{\mathcal{X}}(\mathcal{U}^2,N)\,,$$
$$\mathcal{I}_{\mathcal{X}}(\mathcal{U}^2,N) = \frac{1}{4}\int d^{12}Z\Big\{\mathcal{U}^2\bar{N} + \bar{\mathcal{U}}^2 N - \frac{N\bar{N}}{1-\frac{1}{4}n\bar{n}}\Big\},\ \ n = \bar{D}^4\bar{N}\,.$$

This interaction is $U(1)$ invariant, provided that $\delta_\omega N = -2i\omega\,N$. Thus indeed $S_{\mathcal{X}}(\mathcal{W})$ corresponds to self-dual $\mathcal{N}=2$ system. This action is reproduced after eliminating both superfields $\mathcal{U}$ and N from $\mathcal{S}_{\mathcal{X}}(\mathcal{W},\mathcal{U},N)$. On the other hand, eliminating N,

$$N - (1-\frac{1}{4}n\bar{n})\mathcal{U}^2 + \frac{1}{4}(1-\frac{1}{4}n\bar{n})\bar{D}^4\left[\frac{N\bar{N}\bar{n}}{(1-\frac{1}{4}n\bar{n})^2}\right] = 0\,,$$

we obtain the sought $(\mathcal{U})$ interaction

$$\mathcal{I}_{\mathcal{X}}(\mathcal{U}^2) = \mathcal{I}_{\mathcal{X}}(\mathcal{U}^2, N(\mathcal{U}^2)) = \sum_{n=1}^{\infty}\int d^{12}Z\,\mathcal{L}^{(4n)}_{\mathcal{X}}(\mathcal{U},\bar{\mathcal{U}})\,.$$

A few lowest recursive terms are

$$\mathcal{L}^{(4)}_{\mathcal{X}} = \frac{1}{4}\mathcal{U}^2\bar{\mathcal{U}}^2, \quad \mathcal{L}^{(8)}_{\mathcal{X}} = -\frac{1}{16}\mathcal{U}^2\bar{\mathcal{U}}^2\, A,$$

$$\mathcal{L}^{(12)}_{\mathcal{X}} = \frac{1}{64}\mathcal{U}^2\bar{\mathcal{U}}^2 \left(B\bar{B} + B^2 + \bar{B}^2\right),$$

$$\mathcal{L}^{(16)}_{\mathcal{X}} = -\frac{1}{256}\mathcal{U}^2\bar{\mathcal{U}}^2\Big\{D^4[\mathcal{U}^2(B+2\bar{B})]\bar{D}^4[\bar{\mathcal{U}}^2(\bar{B}+2B)] + (B+\bar{B})(B^2+\bar{B}^2)\Big\},$$

where

$$A := (D^4\mathcal{U}^2)(\bar{D}^4\bar{\mathcal{U}}^2), \quad B := \bar{D}^4 D^4(\bar{\mathcal{U}}^2\mathcal{U}^2), \quad \bar{B} = D^4\bar{D}^4(\bar{\mathcal{U}}^2\mathcal{U}^2).$$

They are capable to restore the original action $S_{\mathcal{X}}(\mathcal{W})$ up to the 18th order upon eliminating $\mathcal{U}, \bar{\mathcal{U}}$. Recently, the action $S_{\mathcal{X}}(\mathcal{W})$ was explicitly given up to the 14th order (Bellucci *et al* 1212.1902 [hep-th]). In the $\mathcal{U}$ language, this corresponds to keeping terms up to the 12th order. The fact that all $\mathcal{U}$ terms above are expressed through the same superfield arguments $A, B, \bar{B}$ makes it probable that the whole interaction $\mathcal{I}_{\mathcal{X}}(\mathcal{U})$ can be written as a sum of the well defined terms related by some general recurrence formula (or even in a closed form).

$(\mathcal{U},\mathcal{W})$ form of the $\mathcal{N}=2$ action up to 10th order

Our aim is to present the auxiliary interaction $\mathcal{I}_{BI}(\mathcal{U})$ which reproduces the $(\mathcal{W},\bar{\mathcal{W}})$ form of the BI action up to the 10th order, i.e. the sum of four terms

$$\hat{I}_{BI} = I^{(4)}_{BI} + I^{(6)}_{BI} + I^{(8)}_{BI} + I^{(10)}_{BI}\,.$$

The corresponding $(\mathcal{U},\mathcal{W})$ BI action is

$$\hat{\mathcal{S}}_{BI} = \mathcal{S}_b + \hat{\mathcal{I}}_{\mathcal{X}} + \hat{\mathcal{I}}_{\mathcal{R}} + \hat{\mathcal{I}}_{\mathcal{Y}}\,, \tag{2}$$

$$\hat{\mathcal{I}}_{\mathcal{X}}(\mathcal{U}) = \frac{1}{4}\int d^{12}Z\,\mathcal{U}^2\bar{\mathcal{U}}^2\left[1-\frac{1}{4}(D^4\mathcal{U}^2)(\bar{D}^4\bar{\mathcal{U}}^2)\right],$$

$$\hat{\mathcal{I}}_{\mathcal{R}}(\mathcal{U}) = \frac{1}{8}\int d^{12}Z\left(-\frac{2}{9}\mathcal{U}^3\Box\bar{\mathcal{U}}^3 + \frac{1}{72}\mathcal{U}^4\Box^2\bar{\mathcal{U}}^4 - \frac{1}{1800}\mathcal{U}^5\Box^3\bar{\mathcal{U}}^5\right),$$

$$\hat{\mathcal{I}}_{\mathcal{Y}}(\mathcal{U}) = \frac{1}{72}\int d^{12}Z\Big[(\mathcal{U}^3\bar{\mathcal{U}}^2 D^4\mathcal{U}^2\Box\bar{D}^4\bar{\mathcal{U}}^3 + \mathcal{U}^2\bar{\mathcal{U}}^3\bar{D}^4\bar{\mathcal{U}}^2\Box D^4\mathcal{U}^3)$$
$$+\frac{1}{2}\mathcal{U}^3\bar{D}^4\bar{\mathcal{U}}^2\Box(\bar{\mathcal{U}}^3 D^4\mathcal{U}^2)\Big].$$

We observe that the first contributions from $\hat{\mathcal{I}}_{\mathcal{Y}}$ appear only in the 10th order. All these $\mathcal{U}$ terms are manifestly $U(1)$ invariant, so the $\mathcal{N}=4/\mathcal{N}=2$ BI action is self-dual up to the 10th order. The road from the $\mathcal{U}$ action to its $\mathcal{W},\bar{\mathcal{W}}$ form is rather complicated technically, nevertheless it is feasible to tread it and to finally obtain the correct $\mathcal{W},\bar{\mathcal{W}}$ action.

Finally, it is instructive to give the explicit form of all orders of the $\mathcal{U}$ interaction $\mathcal{I}_{BI}(\mathcal{U})$ known to date:

$$
\begin{aligned}
&\mathcal{I}^{(4)}_{BI} = \frac{1}{4}\int d^{12}Z\,\mathcal{U}^2\bar{\mathcal{U}}^2\,, \quad \mathcal{I}^{(6)}_{BI} = -\frac{1}{36}\int d^{12}Z\,\mathcal{U}^3\Box\bar{\mathcal{U}}^3,\\
&\mathcal{I}^{(8)}_{BI} = -\frac{1}{16}\int d^{12}Z\left[\mathcal{U}^2\bar{\mathcal{U}}^2 D^4\mathcal{U}^2\bar{D}^4\bar{\mathcal{U}}^2 - \frac{1}{36}\mathcal{U}^4\Box^2\bar{\mathcal{U}}^4\right],\\
&\mathcal{I}^{(10)}_{BI} = \frac{1}{72}\int d^{12}Z\left\{\mathcal{U}^3\bar{\mathcal{U}}^2 D^4\mathcal{U}^2\Box\bar{D}^4\bar{\mathcal{U}}^3 + \mathcal{U}^2\bar{\mathcal{U}}^3\bar{D}^4\bar{\mathcal{U}}^2\Box D^4\mathcal{U}^3\right.\\
&\left.+\frac{1}{2}\mathcal{U}^3\bar{D}^4\bar{\mathcal{U}}^2\Box(\bar{\mathcal{U}}^3 D^4\mathcal{U}^2) - \frac{1}{200}\mathcal{U}^5\Box^3\bar{\mathcal{U}}^5\right\}.
\end{aligned}
$$

We observe that the number of terms in this interaction is drastically smaller as compared with the number of analogous terms in the standard $\mathcal{W}$ representation. This gives a hope that in the $\mathcal{U},\mathcal{W}$ representation it will be possible to somehow sum up all possible terms and so to give a general proof of the self-duality of the $\mathcal{N}=4/\mathcal{N}=2$ BI action.

Summary and Outlook

- **All** duality invariant systems of nonlinear electrodynamics and its $\mathcal{N} = 1, 2$ superextensions admit an off-shell formulation with the auxiliary bispinor fields or their superfield counterparts. This formulation reduces the self-duality constraints to the condition of $U(1)$ invariance of the auxiliary interaction. In the talk it was illustrated by the notorious examples of the Born-Infeld theory and its supersymmetric cousins.
- This universal method was applied for analysis of the self-duality properties of the $\mathcal{N} = 2$ BI action with the nonlinearly realized $\mathcal{N} = 4$ supersymmetry. Its self-duality was proved up to the 10th order in the superfield strengths by constructing the new $(\mathcal{U}, \mathcal{W})$ representation of this action to the same order. This representation contains much less terms compared to the $\mathcal{W}$ form of the action. The conjecture about the general structure of the $(\mathcal{U}, \mathcal{W})$ form of the BI action was put forward.
- As a by-product, the new closed auxiliary-superfield representation was found for the action $S_{\mathcal{X}}(\mathcal{W})$ which is the necessary constituent of the full $\mathcal{N} = 4/\mathcal{N} = 2$ BI action.

- *Some further lines of study*:

  (a) It would be interesting to inquire whether the 10th order-truncated $(\mathcal{U}, \mathcal{W})$ BI action can be somehow promoted to all orders in the auxiliary superfields $\mathcal{U}, \bar{\mathcal{U}}$, thus providing the $(\mathcal{U}, \mathcal{W})$ form of the complete $\mathcal{N} = 4/\mathcal{N} = 2$ BI action

  (b) The closely related problem is to understand how the hidden spontaneously broken $\mathcal{N} = 4$ supersymmetry (including the central charge transformations) is realized in the $(\mathcal{U}, \mathcal{W})$ formulation, i.e. on the extended superfield set $\mathcal{W}, \bar{\mathcal{W}}, \mathcal{U}, \bar{\mathcal{U}}$. This problem stays in the case of $\mathcal{N} = 1$ BI action as well.

  (c) It would be tempting to find some general framework for the "irregular" $\mathcal{Y}$ terms of the $\mathcal{N} = 2$ BI action. They are present starting from the 10th order and, perhaps, could be understood as a recursive solution of some nonlinear superfield equation like that for the auxiliary superfield $\mathcal{X}$.

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